J Geod (2005) 79: 24–32
DOI 10.1007/s00190-005-0439-0

ORIGINAL ARTICLE

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Multi-reference evaluation of uncertainty in earth orientation parameter measurements

Received: 25 February 2004 / Accepted: 17 January 2005 / Published online: 18 April 2005

Abstract Uncertainties in polar motion and length-of-day measurements are evaluated empirically using several data series from the space-geodetic techniques of the global positioning system (GPS), satellite laser ranging (SLR), and very long baseline interferometry (VLBI) during 1997–2002. In the evaluation procedure employed here, known as the *three-corner hat* (TCH) technique, the signal common to each series is eliminated by forming pair-wise differences between the series, thus requiring no assumed values for the "truth" signal. From the variances of the differenced series, the uncertainty of each series can be recovered when reasonable assumptions are made about the correlations between the series. In order to form the pair-wise differences, the series data must be given at the same epoch. All measurement data sets studied here were sampled at noon (UTC); except for the VLBI series, whose data are interpolated to noon and whose UT1 values are also numerically differentiated to obtain LOD. The numerical error introduced to the VLBI values by the interpolation and differentiation is shown to be comparable in magnitude to the values determined by the TCH method for the uncertainties of the VLBI series. The TCH estimates for the VLBI series are corrupted by such numerical errors mostly as a result of the relatively large data intervals. Of the remaining data sets studied here, it is found that the IGS Final combined series has the smallest polar motion and length-of-day uncertainties.

Keywords Earth rotation · Measurement error · Combination

1 Introduction

Earth orientation parameters (EOP) are routinely measured by various techniques including the global positioning system (GPS), satellite laser ranging (SLR), and very long baseline interferometry (VLBI). The objective combination of multiple data sets by methods such as weighted least squares and Bayesian statistical (Kalman filter) estimation requires some quantification of the relative accuracy of these measurements. Statistical sampling of the measurement error, typically an empirical evaluation of variances and covariances, is usually dependent on some assumed values for the ground-truth EOP. In practice (e.g., Gross et al. 1998; Fernandez et al. 2001; Gambis 2002), particular multi-measurement combinations are often designated as proxies of the ground truth. Also, in the so-called "Allan Variance" approach (e.g., Gambis 2002), the proxy ground truth is effectively a smoothed version of the measurement series itself, where the smoothing is designed to discriminate among some frequency-dependent components in the error process. An obvious shortcoming of these error estimation techniques is that the true signal to which the error is referenced is usually not known exactly. A technique that does not require ground-truth values is thus desirable as an alternative and independent source of EOP measurement error statistics.

When multiple independent measurements are available for the same signal, the variance and covariance can be evaluated directly from the measurement data sets under certain algebraic assumptions. Weiss and Allan (1986) have used such a technique, the so-called *three-corner hat* (TCH) technique, to determine the accuracy of GPS clocks. In particular, to estimate the stability of a single clock, measurements are made by three clocks so that differences in pairs of measurements can be determined. The variance in each clock is then evaluated from these pair-wise differences under the assumption that the noise processes of the three clocks are independent. The International Earth Rotation Service (IERS) was using a generalized version of the TCH method, applicable to more than three sets of data, to obtain independent estimates of EOP measurement error (e.g., Gambis et al. 2000).

The TCH method is algebraically straightforward to apply to exactly three data sets that have statistically independent measurement error processes. When there are more than three sets of data as in the case of the EOP measurements,

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Table 1 Data sets used in the evaluation

Data set	Label	Sampling
Jet Propulsion Lab, Quick-look GPS	GPS.JQ	Daily (noon)
International GPS Service, Rapid combined	GPS.IR	Daily (noon)
International GPS Service, Final combined	GPS.IF	Daily (noon)
Joint Center Earth System Technology, SLR	SLR	Daily (noon)
International VLBI Service, combined multibaseline	VLBI.IM[b]	Every 2.9 days[a]
Goddard Space Flight Center, multibaseline VLBI	VLBI.GM[b]	Every 2.7 days[a]
Goddard Space Flight Center, intensive VLBI	VLBI.GI[b,c]	Every 1.8 days[a]
Atmospheric (NCEP) & oceanic (ECCO) angular momentum	O+AAM[c]	Daily (midnight)
International Earth Rotation Service, C 04	C04	Daily (midnight)
Jet Propulsion Lab, SPACE2002	SPACE	Daily (noon)

[a] average time interval
[b] LOD obtained by differentiation of UT1 values
[c] PM values not available

however, application of TCH requires some additional technical considerations. One issue concerns correlation among the measurement errors. For example, except for the JPL "GPS Quick-look" series, the GPS measurements of the Earth rotation rate (length of day, or LOD) are referenced to Universal Time (UT1) values measured by VLBI; the error processes of these two measurement types can thus be reasonably expected to contain a common component and hence be correlated. While the key mathematical assumption of the TCH method is the statistical independence of the error processes, a generalized version of TCH does allow some of the data sets to have correlated uncertainties. In this paper, we examine mathematically how measurement sets with correlated noise can be admitted to the generalized TCH procedure.

Another issue is the alignment of the data sets. Since the first step of the TCH method is the computation of the difference between *every* pair of measurements, all the data sets must be aligned in time. The GPS and SLR data sets studied here are given at noon (UTC) at daily intervals. On the other hand, the VLBI data are given at irregular epochs at an average interval of 2–3 days (Table 1). Thus, the VLBI measurements must be interpolated to the nearest noon epoch and the GPS and SLR measurements must be subsampled to the epochs of the interpolated VLBI measurements. The issue is further complicated by the fact that VLBI measures UT1 whereas the satellite techniques of GPS and SLR measure LOD, which is the time derivative of UT1. While UT1 is the quantity of interest in spacecraft navigation and in coordinate transformation between the terrestrial and celestial reference frames, LOD is often the quantity directly relevant to scientific studies of the Earth's rotation (e.g., Rosen and Salstein 1983; Eubanks et al. 1985; Hide and Dickey 1991). Here, the VLBI UT1 measurements were numerically differentiated to recover LOD. The VLBI data sets were thus both interpolated and differentiated. This paper shows empirically that such interpolation and differentiation procedures can introduce a significant level of numerical errors in the TCH evaluation. We will also show that despite such errors, the VLBI measurements still contribute to the necessary algebraic references over which the uncertainties of other data sets are established and hence play a crucial role in the TCH computation.

2 Measurements and preprocessing

The EOP variables considered here are the LOD and the x and y components of polar motion (PM). Seven measurement series spanning March 1997 to December 2002 are used. These include three GPS (labeled as: GPS.JQ, GPS.IR, GPS.IF), one SLR (SLR), and three VLBI (VLBI.IM, VLBI.GM, VLBI.GI) data sets from various analysis centers as summarized in Table 1 (top group). The data sets are preprocessed by removing long-period solid Earth and ocean tidal effects, outliers (four times standard deviation), and constant offsets and rates (see Gross et al. 1998 for details). Since VLBI.GI does not report PM values, the evaluation of PM errors is performed with the remaining six data sets. Also, for the PM evaluations, only the data after 1 March 1998 are used due to noticeably larger scatter in some GPS-based PM values before that date. Advances in instrumentation and calibration lead to improvements in measurement accuracy over time, the effect of which can be more pronounced in some data sets than others; however, we will assume stationarity of the error statistics over the evaluation period.

Because LOD variations are known to be caused largely by changes in atmospheric angular momentum (e.g., Rosen and Salstein 1983; Hide and Dickey 1991), we include an angular momentum data set (O+AAM) in the evaluation as a proxy LOD data set. The O+AAM data set is the sum of global angular momentum series computed at daily intervals from simulations of ocean and atmospheric general circulation models. The atmospheric component is the sum of the wind and inverted barometer surface pressure effects and is obtained from the (US) National Center for Environmental Prediction reanalysis, while the oceanic component is the sum of the current and bottom pressure effects and is from the Jet Propulsion Laboratory (JPL) ECCO data-assimilative model (version kf052d). A linear trend was removed from the O+AAM data (in such a way that the difference with the GPS.IF data is minimized). By inclusion of the O+AAM series, the number of LOD data sets becomes eight.

In addition to the data sets described above, we also examine the error statistics of two combination products: the IERS "C 04" (C04) and JPL "SPACE2002" (SPACE) series. These combination series are treated separately from the other eight

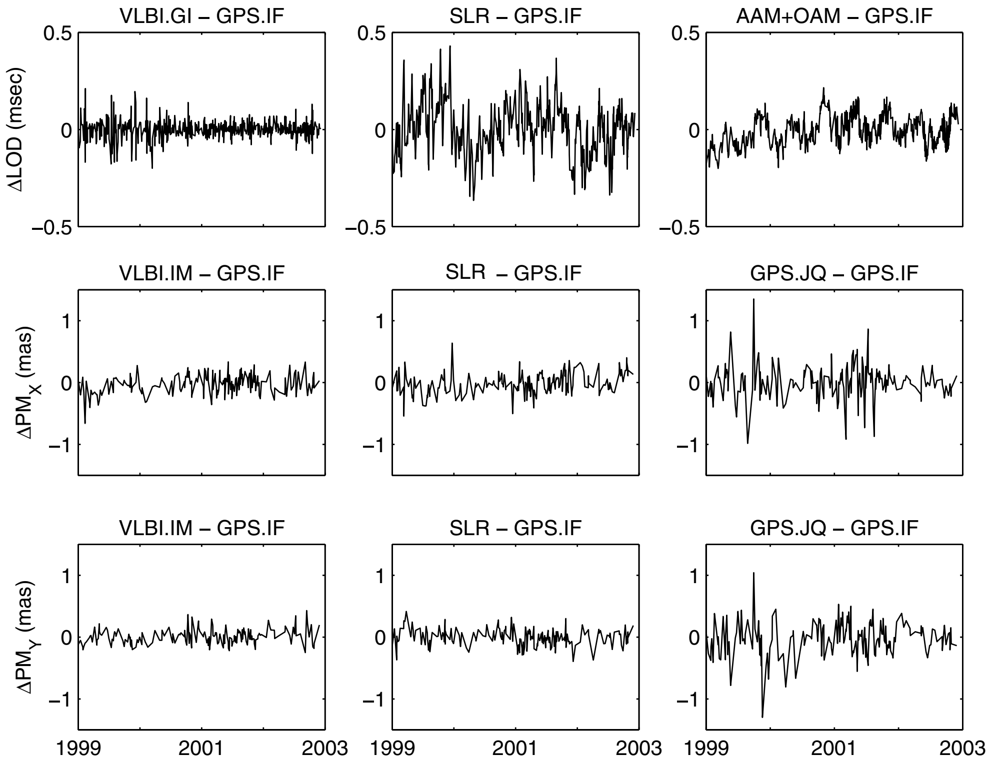

Fig. 1 Examples of difference processes (differences between pairs of measurements) used in the TCH evaluation

series because they include, and hence are correlated with, data from all instrument types (GPS, SLR, VLBI).

Our TCH evaluations are based on values given at noon (UTC) that were interpolated and subsampled to common epochs. The C04 and three VLBI data sets are not given at noon and are thus subject to interpolation error. In addition, the UT1 values of VLBI series are converted to LOD values by numerical differentiation, thus subjecting them to additional differentiation error. Interpolation and differentiation are accomplished with a smoothing B-spline scheme (Inoue 1986; Chin et al. 2004) which allows analytic differentiation and yields more accurate results than a linear interpolation followed by a center-differencing. Since the GPS and SLR series are simply subsampled to the epochs of the interpolated VLBI series, they are not subject to either interpolation or differentiation error. The daily C04 and O+AAM series are interpolated to noon using the same B-spline interpolation scheme (without differentiation) and then subsampled to the epochs of the interpolated VLBI series. The SPACE series, whose values are given at noon, were simply subsampled to the epochs of the interpolated VLBI series.

3 Generalized three-corner hat method

Let the measured values be y_i, $1 \leq i \leq N$, where the index i specifies a particular data set and N is the number of data sets. We consider the standard, additive-noise model for measurement processes as

$$y_i = x + w_i \,, \tag{1}$$

where x is the true signal and w_i is a zero-mean white noise process representing the measurement error. This model is routinely used in Kalman filtering and least-squares processing of measurements. The pair-wise difference among the measurements y_i eliminates the common signal x; we define

$$z_{ij} \;\equiv\; y_i - y_j \;=\; w_i - w_j \,, \tag{2}$$

for $i < j$ and refer to z_{ij} as the *difference process*. Figure 1 shows examples of difference processes used in this study. The correlation among the difference processes can constrain the statistics of the white noise processes as

$$R_{ik} + R_{j\ell} - R_{i\ell} - R_{jk} = \langle z_{ij} z_{k\ell} \rangle \,, \tag{3}$$

where $R_{ij} \equiv \langle w_i w_j \rangle$, $i \leq j$, are the noise covariances and the angular brackets denote a time average.

We wish to compute the covariances R_{ij} from the empirical correlations $\langle z_{ij} z_{k\ell} \rangle$ that are evaluated from the measurements. Given N measurements, the number of distinct covariances R_{ij} is $(N+1)N/2$. Since only $N-1$ linearly independent differences z_{ij} are available, the number of effective constraints (Eq. 3) is only $N(N-1)/2$, which is the number of distinct auto- and cross-correlations among the $N-1$ differences. The unknowns out-number the constraints by N. Thus, N additional equations are needed to supplement Eq. (3). A key assumption of the TCH technique is that N (or more) of the cross-correlations are zero, or

$$R_{ij} = 0 \,, \tag{4}$$

for some i and j where $i \neq j$. For example, in a classic application of the TCH technique, there are exactly three *statistically-independent* measurement sets, i.e., $N = 3$ and $R_{12} = R_{13} = R_{23} = 0$. The three variances R_{ii}, $1 \leq i \leq 3$, are then uniquely constrained by Eq. (3). In a generalized case where $N > 3$, the number of cross-correlation $N(N-1)/2$ is larger than N, and more than N assumed constraints (Eq. 4) could hence be applicable depending on the physical scenario that admits such assumptions. Thus, for the generalized cases, the unknown R_{ij} can be over-constrained by the total set of the measurement (Eq. 3) and assumed independence (Eq. 4) equations. In practice, pseudo-inversion (e.g., Golub and van Loan 1989) can be used to solve the over-constrained set of equations.

3.1 Assumption of independence

There can be some apparent correlation among the noise processes even if the measurements are physically unrelated. The TCH technique can fail when the assumption of independence among the measurements is not satisfied empirically. For example, in the classic case of $N = 3$, the variances are computed as

$$R_{ii} = \langle y_i^2 \rangle + \langle y_j y_k \rangle - \langle y_i y_j \rangle - \langle y_i y_k \rangle \,, \tag{5}$$

where i, j, k are *distinct* signal indices. The right hand side of Eq. (5) can conceivably become negative depending on the observed correlation values, demonstrating that the TCH method has no inherent algebraic guarantee for the variance values to be positive. On the other hand, if the independence constraint Eq. (4) is correct, the last three terms of Eq. (5) become equal to $-\langle x^2 \rangle$, and the correct solution can be expected. Correctness of the independence assumption Eq. (4) is thus important to the success of the TCH method. This point can be demonstrated numerically using simulated noise processes. The TCH method is applied to a triplet ($N = 3$) of simulated noise processes which have been generated using zero-mean Gaussian distributions with a given prescribed value for the correlations among them; this procedure is repeated 400 times for each correlation value. As discussed earlier, TCH must assume that each pair among the triplets is uncorrelated. Figure 2 displays a plot of the average error in the TCH variance estimate as a function of the correlation value. The error increases almost linearly with the correlation coefficient. When the actual correlation is zero as assumed in Eq. (4), the average error is found to be approximately 1%, which is the order of the accuracy of the random number generator used in the simulation.

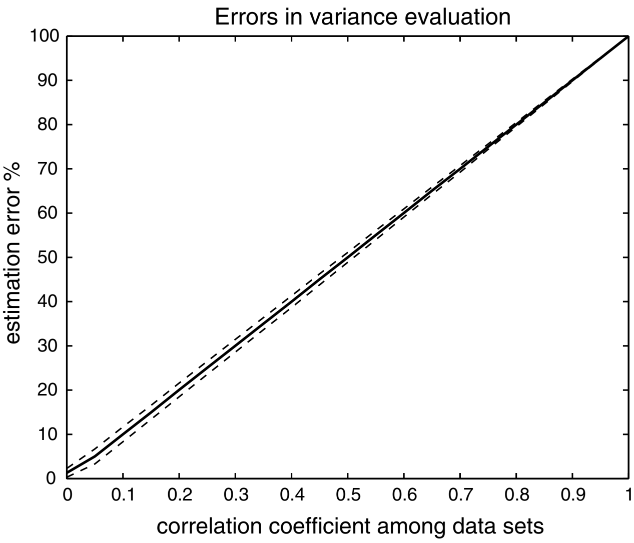

Fig. 2 Percent errors in variances computed with the three-corner hat method, as a function of the correlation coefficient among the data sets. The *solid line* is the mean of 400 simulations, while the *dashed lines* are the corresponding single-standard-deviation envelops

3.2 Choice of independent pairs

For $N > 3$, at least N cross-variances R_{ij} must be chosen among the $N(N-1)/2$ candidates for application of the independence constraint Eq. (4). These choices are usually made based on physical insights. For example, the errors in GPS and SLR can be assumed to be uncorrelated because the instrumentation and principle behind the two measurement types are largely unrelated. On the other hand, the errors in the GPS-based data from different analysis centers should be assumed to be correlated since the same instrument is common to all the data sets. In addition, there are algebraic restrictions, one of which is as follows:

Lemma 1 (a necessary condition for uniqueness)**.** *The TCH solution is unique only if every noise process is assumed uncorrelated to at least one other noise process.*

Algebraic necessities such as this need to be matched with physical reasoning since an erroneous assumption of independence can lead to failure of the TCH technique as demonstrated previously. For the eight non-combination data sets (first 8 rows, Table 1), we assume that the noise processes are independent across the three instrumentation groups of GPS, SLR, and VLBI, while non-zero correlations are allowed within each group (Fig. 3). When N is 3 or 4, Lemma 1 is satisfied automatically since at least N pairs must be assumed uncorrelated, as mentioned earlier.

To prove Lemma 1, let us write Eqs. (3) and (4) in matrix form as $\mathbf{Ar} = \mathbf{b}$, where $\mathbf{r}$ is a column vector of the $(N+1)N/2$ unknown covariances R_{ij}. Without loss of generality, we define the indices (i, j) of z_{ij} and R_{ij} so that $i \leq j$. We assume that an arbitrary noise process w_1 is correlated with all other noise processes. We then wish to show that $\mathbf{A}$ is rank-deficient, so that $\mathbf{r}$ cannot be determined uniquely. Since Eq. (4) does not apply to the covariances R_{1j} for any j, all constraint equations involving these covariances must be derived from Eq. (3) in either of the following two forms:

$$R_{11} - R_{1j} - R_{1i} - R_{ij} = \langle z_{1i} z_{1j} \rangle$$
$$R_{1k} - R_{1j} - R_{ki} - R_{ij} = \langle z_{1i} z_{kj} \rangle \,.$$

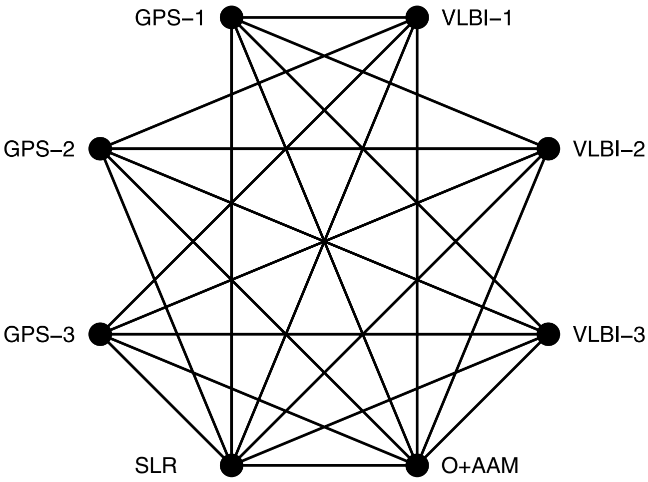

Fig. 3 Application of independence constraints among the eight measurement data sets. A pair of data sets connected by a straight line is assumed to be uncorrelated (with respect to the noise processes) during TCH evaluation

From these, the following structure can be observed in matrix **A**: for any row of **A**, the sum of elements at the column positions associated with R_{1j} for $2 \leq j \leq N$ is either -2 or 0. When this sum equals -2, the element associated with R_{11} is 1, while the element is 0 when the sum is 0. Thus, for any row of **A**, the sum $(R_{1j}, 2 \leq j \leq N)$ is always -2 times the element (R_{11}). This implies that the matrix columns associated with R_{1j} for $1 \leq j \leq N$ are linearly dependent, indicating that **A** is rank-deficient.

3.3 Comparison to combination products

A consequence of Lemma 1 is that TCH evaluation is not suitable for the combination products, including C04 and SPACE, which tend to contain data from most available instruments and hence are correlated to all measurement types. Since the uncertainty in the combined values is dependent on the noise process associated with each assimilated measurement, it is not appropriate to assume the necessary condition in Lemma 1 to hold. Evaluation for the C04 and SPACE data sets is thus accomplished here by applying the TCH technique *indirectly* as follows: first, a maximum likelihood estimate $\widehat{x}$ of the true signal x is computed using all the other (those which are not combined products) data sets along with their associated covariances that were estimated using the TCH method as described above. The estimate $\widehat{x}$, referred to as the *TCH-combination* hereafter, is then used as the reference time-series for evaluation of uncertainties in the C04 and SPACE series. Specifically, let **R** be the covariance matrix whose (i, j)th element is R_{ij} $(R_{ji} = R_{ij})$. Application of the standard maximum-likelihood formulation (e.g., Lewis 1986) to Eq. (1) then yields

$$\widehat{x} = (\mathbf{h}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{h})^{-1}\mathbf{h}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{y}\,, \tag{6}$$

where **y** and **h** are N-dimensional vectors of the measurements y_i and ones (1's), respectively, with the superscript $^{\mathrm{T}}$ denoting the transpose. The mean-square of the differences $y_i - \widehat{x}$, where i identifies either the C04 or SPACE series, are then computed as the estimates of the variances of the combination products.

4 Results

4.1 Variances

Table 2 displays the standard deviations of the measurement noise processes obtained using the TCH method. The noise values of the LOD series range approximately between 15 and 135 μs while the variability of the LOD signal is approximately half to third of a milli-second. The noise values of the x and y components of the PM (PM-X and PM-Y, respectively) series range between 0.050 milli-arc-second and 0.500 milli-arc-second (mas), while the variability in each of the signal components is on the order of 100 mas.

4.1.1 Effect of interpolation and differentiation

As described in Sect. 2, some of the data sets are subjected to interpolation (and differentiation, in the case of the VLBI UT1 series) for time alignment. The effect of the interpolation procedure on the TCH evaluation is examined through simulation experiments. The SPACE2002 LOD series, available at both noon and midnight, is used as the ground truth and is subsampled to simulate the gaps in the actual data sets. Numerical integration of the SPACE2002 LOD values is performed to simulate the VLBI UT1 series. The simulated data sets are then subjected to the same interpolation (and differentiation) routine as that used in the TCH evaluation. The resulting interpolated values are finally compared to the ground truth values. The numerical error introduced by interpolation is found to be negligible, less than 1 μs on the average, when the simulated gaps have a duration of a day (for the cases where data sampled at midnight must be interpolated to noon epochs). For the simulated VLBI series which contain much larger data gaps, however, the error is found to be considerably larger at 14–25 μs (column *ID*, Table 2). Here, the interpolation procedure is found to introduce five to six times more error in magnitude than the differentiation procedure.

The numerical error introduced by interpolation (and differentiation) of the VLBI measurements is typically comparable in magnitude to the corresponding values determined by the TCH method (Table 2). Interpolation procedures can unintentionally low-pass filter the signal, and such distortion can be severe especially for the VLBI series whose data gaps can be as long as two weeks. The power spectral densities of the VLBI LOD series (interpolated and differentiated from the UT1 series) are indeed found to lack the high frequency components evident in the spectral densities of the other LOD measurement series. Specifically, the spectra from the VLBI data deviate more significantly from the $1/f^2$ power-law model (Eubanks et al. 1985) than do the other spectra, by under-shooting the model at frequencies above 0.1 cycles

Table 2 Standard deviations of the measurement errors estimated using the TCH method. Uncertainties in the estimates due to interpolation and differentiation errors (*ID*) and violation of the stationarity assumption (*SA*) have also been determined empirically

Data set	LOD (μs)			PM (mas)					
	(Noon)	*ID*	*SA*	PM-X	*ID*	*SA*	PM-Y	*ID*	*SA*
GPS.JQ	38.9		±3.4	0.301		±0.035	0.240		±0.033
GPS.IR	20.2		±2.6	0.134		±0.030	0.071		±0.018
GPS.IF	14.5		±0.7	0.080		±0.021	0.049		±0.024
SLR	133.0		±14.8	0.171		±0.016	0.150		±0.023
VLBI.IM	26.3	±24.5	±2.7	0.127	±0.281	±0.008	0.122	±0.192	±0.023
VLBI.GM	25.3	±22.5	±3.5	0.522	±0.269	±0.188	0.295	±0.202	±0.126
VLBI.GI	47.4	±14.4	±12.1						
O+AAM	78.3		±12.1						

Table 3 Standard deviations of the LOD measurement errors for the low-passed filtered data sets (see text)

Smoothed data set	LOD (μs)
GPS.JQ	18.7
GPS.IR	10.4
GPS.IF	9.1
SLR	113.0
VLBI.IM	11.5
VLBI.GM	10.0
VLBI.GI	10.3
O+AAM	71.2

per day. To assess the relative accuracy of the VLBI measurements with respect to the other measurements, the TCH computation was repeated after all LOD data sets are *intentionally* low-pass filtered to remove the discrepancies found at the high frequencies described above. The (intentional) low-pass filter has a cut-off frequency of 0.14 cycles per day, designed to equalize the spectral contents of all data sets. The resulting TCH values are displayed in Table 3. While these are not estimates of actual measurement errors, the differences between the VLBI and GPS values are much smaller than those in Table 2.

This further demonstrate that the TCH estimates for the VLBI measurement uncertainties shown in Table 2 are dominated by interpolation and differentiation errors.

4.1.2 Effect of stationarity

The columns labeled *SA* in Table 2 represent an evaluation of errors due to violation of the stationarity assumption. These *SA* values are computed as the variability among the TCH estimates when different time-segments of the data sets are used. The full data sets are divided equally into four subsegments, the TCH evaluations are performed on each subsegment, and the variability among the estimated variance values is computed. The range of uncertainty due to violation of stationarity assumption is approximately 10–25% of the total estimated uncertainties for both LOD and PM.

4.2 Comparison to independent estimates

The TCH estimates for the measurement error (standard deviations) are compared to the corresponding prior statistics used in the SPACE2002 combination procedure. The measurement error evaluation procedure used in SPACE2002 (Gross 1996, 2000) is different from the one used in TCH. Briefly, the stated uncertainties of each data set used in SPACE2002 were adjusted by a scale factor whose value is such that the residual of each series, when differenced with a combination of all others, has a reduced chi-square (e.g., Bevington 1969) near one. During the iterative procedure that is used to determine the scale factor, outliers were discarded as described in Sect. 2.

Figure 4 displays the uncertainties of the GPS.IF and SLR data sets as estimated by the TCH method and the SPACE2002 uncertainty adjustment procedures. The standard deviations from the two evaluation procedures are in good agreement with each other for PM (both components) in the GPS.IR and VLBI.IM data sets as well as those displayed in Fig. 4. For the remaining PM data sets (GPS.JQ and VLBI.GM) and all the LOD data sets, the TCH estimates are higher than the SPACE2002 estimates.

4.3 Correlation coefficients

The TCH evaluation performed here is essentially a solution of 26 linear equations given by Eq. (3) and Eq. (4) for 21 unknowns R_{ij}. Since the unknowns are over-constrained by the equations, the solution is obtained by Moore-Penrose ("pseudo") inversion (e.g., Golub and van Loan 1989), which minimizes the difference between the left and right hand sides in the least-squares sense. Due to this least-squares procedure, the recovered correlation coefficients are not strictly zero, even for those coefficients specified to be zero by application of the independence assumption Eq. (4). For example, the evaluated correlation coefficients between SLR and the other data sets are only approximatedly zero (see Tables 4 and 5).

For correlation among the LOD uncertainties (Table 4), the two multibaseline VLBI analyses VLBI.IM and VLBI.GM show strong correlation (0.88). Positive correlations can also be observed among the GPS-based analyses but at much lower magnitudes. Smoothing caused by the interpolation procedure may contribute to the high positive correlation between the VLBI.IM and VLBI.GM uncertainties; however, such an effect of interpolation on the correlation values appears to be relatively small because these two VLBI series

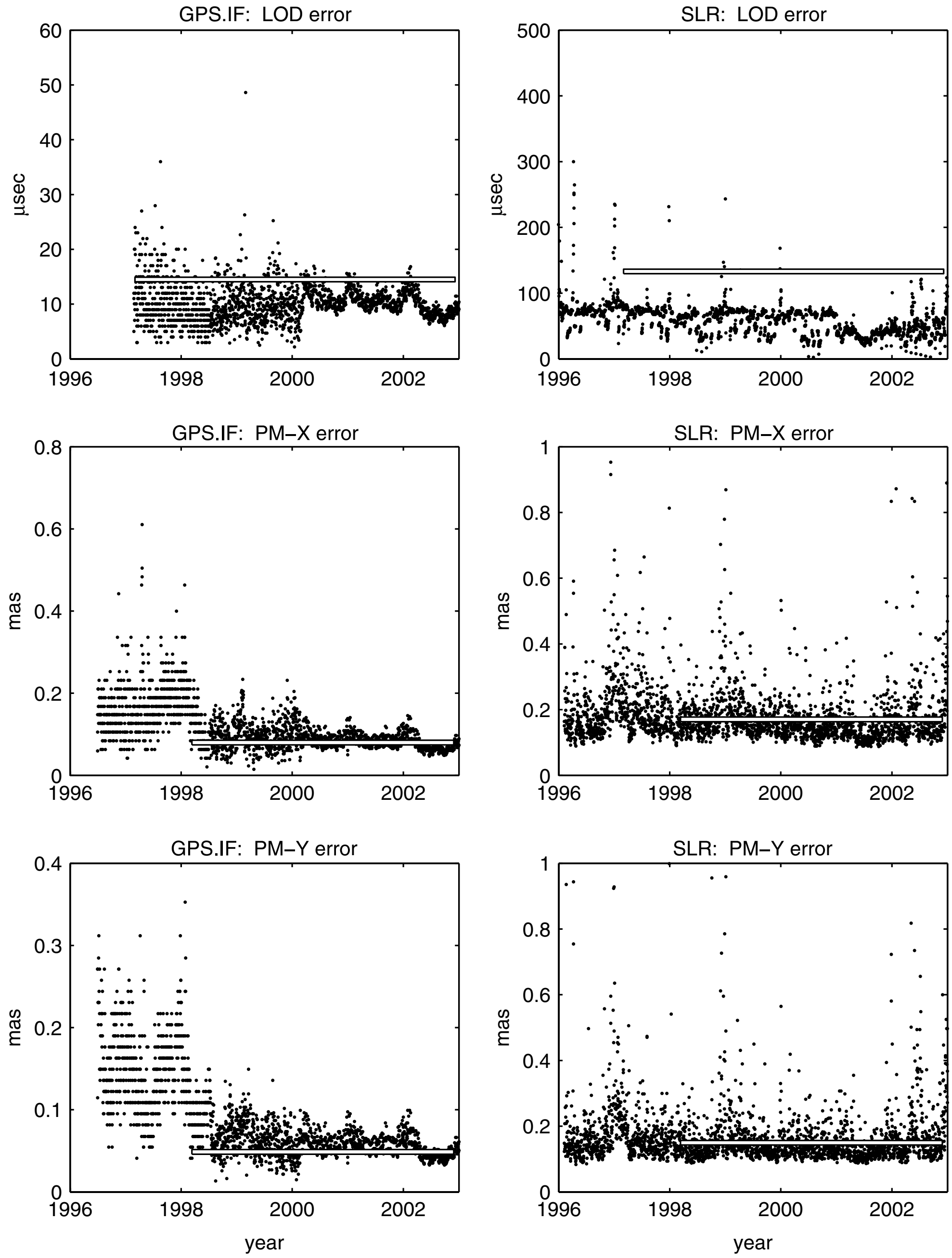

Fig. 4 The root-variance values computed using the TCH method (*light horizontal lines*) and corresponding values used as prior parameters in the SPACE2002 combination procedure (*dots*)

Table 4 Correlation coefficients among the LOD data sets, evaluated with the TCH method

LOD data set	GPS.IR	GPS.IF	SLR	VLBI.IM	VLBI.GM	VLBI.GI	O+AAM
GPS.JQ	+0.28	+0.16	+0.03	−0.02	−0.03	−0.02	0.00
GPS.IR		+0.37	+0.01	−0.02	−0.05	−0.05	0.00
GPS.IF			−0.03	+0.06	+0.04	+0.01	−0.06
SLR				0.00	0.00	+0.01	0.00
VLBI.IM					+0.88	+0.11	0.00
VLBI.GM						+0.10	0.00
VLBI.GI							−0.02

Table 5 Correlation coefficients among the PM data sets, evaluated with the TCH method

PM data set	GPS.IR		GPS.IF		SLR		VLBI.IM		VLBI.GM	
	X	Y	X	Y	X	Y	X	Y	X	Y
GPS.JQ	+0.50	+0.08	+0.39	−0.42	−0.09	−0.09	+0.02	−0.09	+0.02	+0.09
GPS.IR			+0.67	−0.09	+0.12	+0.13	+0.05	−0.07	−0.05	−0.04
GPS.IF					+0.12	+0.27	−0.06	+0.09	−0.03	−0.17
SLR							−0.05	+0.15	+0.01	−0.06
VLBI.IM									+0.29	+0.25

Table 6 Root-mean-squares difference between each of the multi-measurement combination products, C04 and SPACE, and a TCH-based maximum likelihood combination of the measurements

Data set	LOD (μs)		PM (mas)			
	(Noon)	*SA*	PM-X	*SA*	PM-Y	*SA*
C04	22.1	±2.1	0.086	±0.011	0.093	±0.013
SPACE	17.6	±1.7	0.047	±0.014	0.066	±0.023

show much lower correlations with VLBI.GI, which was subjected to the same interpolation procedure.

For the PM components (Table 5), the correlation between the two multibaseline VLBI analyses VLBI.IM and VLBI.GM is significantly lower than between their LOD components. The correlation structure among the GPS-based analyses shows some differences between the x and y components. While the three GPS-based analyses show positive correlations for PM-X, correlations for PM-Y are non-zero, or negative (-0.42) in the case between GPS.JQ and GPS.IF.

4.4 Correlation between GPS and VLBI

As discussed previously, the TCH evaluation requires an assumption that selected pairs of the noise processes are independent. In our evaluation, the measurement errors in different instrumentations (GPS, SLR, VLBI, and numerically simulated atmospheric and oceanic circulations) were assumed to be mutually independent. This assumption may not be exactly true for all such data pairs. In particular, the analysis procedures for the GPS.IR and GPS.IF LOD data are referenced partially to VLBI measurements, implying correlations between these GPS and VLBI series. We have hence repeated the TCH evaluation for LOD *without* assuming independence between each of the two GPS and every VLBI series. The standard deviation values from this new TCH evaluation are within a single micro second of the corresponding values shown on Table 2. The only exception is the value for GPS.IF, which has increased by about 5 μs in the new evaluation, an amount comparable to the inherent numerical uncertainty in the TCH method. Similarly, the correlation coefficient values did not change much in the new calculation (cf. Table 4), except for the correlations between the GPS.IF uncertainties and each of the VLBI.IM and VLBI.GM uncertainties, which increased to 0.23 and 0.21, respectively.

4.5 Comparison of combination products

Table 6 displays the root-mean-squares differences between the TCH-combination (Eq. 6) and each of the C04 and SPACE combinations. For each EOP (LOD, PM-X, PM-Y), these differences among the combination series are smaller than most of the measurement uncertainties (Table 2), indicating a relatively good agreement among the combination series and an evidence that the result of our TCH evaluation are consistent (at least not in an obvious disagreement) with independently computed combined products.

Since not all of the measurements considered in this study were incorporated into the C04 and SPACE combinations (e.g., GPS.IR and VLBI.GM are not incorporated into SPACE), some measurement time-series have smaller mean difference to the TCH-based combination than the other two combinations. In particular, the GPS.IF measurement error estimates (Table 2) are lower than most of the corresponding values for the C04 and SPACE mean-differences.

5 Discussion and conclusion

The TCH approach was used to estimate the uncertainty in the Earth orientation measurement series determined by the space-geodetic techniques of SLR, VLBI, and GPS. The results of the TCH evaluation presented here appear to be in reasonable agreement with independent estimates. Nevertheless, we have shown empirically that our results do not provide a fair assessment of the uncertainty in the VLBI-based measurements due to additional uncertainty introduced by the interpolation procedure. It is expected that such interpolation error could be significantly reduced if VLBI data were made available at a daily frequency. Note that VLBI data sets are crucial for the TCH method because it is one of the three widely available instrument types, along with GPS and SLR, for EOP observation. Given our premises that the noise processes from a single instrument are mutually correlated (e.g. among all GPS data sets) and that each instrument type is independent from another, Lemma 1 requires at least three instrument types for TCH evaluation problem to be well-posed.

All the series studied here are nominally given within the same terrestrial reference frame, namely, ITRF2000.

However, since each series was determined using a different ground station network, each series is actually given within its own terrestrial reference frame that was tied, with some uncertainty, to ITRF2000. Furthermore, since the individual stations that comprise each network can change from one observing session to another, the values in an individual series can be given within different terrestrial reference frames. Because changes in the Earth's orientation degenerate with changes in the orientation of the terrestrial reference frame, differences between reference frames can introduce differences in the Earth orientation parameters. Thus, the EOP uncertainties recovered here include contributions due to differences in the reference frames within which the measurements are given. Removing the influence of reference frame differences on EOP combinations can be done by the rigorous approach of jointly combining reference frames and EOPs (J. Ray, personal communication, 2003). Using such an approach with the TCH method would likely yield somewhat smaller estimates for the EOP uncertainties. From this perspective, the results presented here represent upper bounds on the true uncertainties of the EOP measurements.

Acknowledgements This research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration (NASA).

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